The Transit of Venus 2004*

– Observation and Measurement of the Sun's Parallax –
(preliminary version)

Udo Backhaus, University of Duisburg-Essen, Germany

October 25, 2005

The Observation and measuring of Venus transits in front of the Sun have for a long time been the best opportunity to determine the distance between Earth and Sun. Even if the Astronomical Unit has today been determined much more precisely the transit of June 8th, 2004, which will be perfectly observable on the longitude of Germany, will offer a extraordinary opportunity to retrace the historical measurements by employing modern methods and to practise the cooperation between schools, amateurs and universities.

Contents

*Thanks to my brother, Prof. Horst Backhaus, not only for smoothing my rough English but also for valuable didactic hints.

1 Introduction

On June 8th, 2004, Venus, observed from Earth, will pass the Sun's face from east to west. A so-called transit of Venus is one of the rarest astronomical events that can be predicted precisely. No living human being has so far observed such an event, simply because none happened during the previous century. And only on June 6th, 2012, we will be offered another opportunity to observe Venus in front of the Sun[1]. In fact, only five transits of Venus have been observed and described by human beings so far (1639, 1761, 1769, 1872 and 1882). Transits of Venus have played an important role in the development of modern astronomy because their measurements provided for the most precise measure of the Sun's distance until the end of the nineteenth century.

2 The Astronomical Unit

In the 18th and 19th century several expeditions were equipped and launched into various distant regions of the world with the objective to provide astronomers with suitable

[1]However, we then will have to make a trip to the other side of the globe because at that time it will be night in Europe.

observation data from the very rare Venus transits. Why did the governments of so many countries spend so much money and why were the astronomers willing to endure the harsh conditions of those expeditions ([14]) to obtain a better measure of the Sun's distance? And why is it still important to know not only the value of the Astronomical Unit but also the methods which enabled a more and more precise determination?

These tremendous efforts of governments and human beings express a large interest in the distance to the Sun. The reasons for that are scientific as well as economic:

- " The distance to the Sun is the basic unit for determining the size of the whole solar system: Knowing the distance to the Sun, we can determine the size of the whole solar system. Since the introduction of the heliocentric world view it has been rather easy to determine the radii or the maior semi-axis of the planets' orbits. However, the results of these measurements reveal all the distances as multiples of the radius of the Earth's orbit. And the value of that was not very well known. For example, the doubling of the distance between Earth and Sun would have resulted in a respective enlargement of the whole solar system. But the ancient value of the Sun's distance which was used until the seventeenth century was in fact too small by a factor of 20.

- The distance to the Sun also forms the basis for measuring the distance to the stars.

- Knowing the distances in the solar system it is possible to determine the astrophysical properties of the Sun and the planets. For instance, the size of the Sun and of the planets can be deduced from their apparent size. The mass of the Sun can be calculated by the help of the law of gravity, provided that the gravitational constant is known. The radiation power of the Sun can be derived from the solar constant measured on Earth.

- " As a crucial precondition for a growing worldwide sea traffic, a more precise astronomical navigation had to rely on more precise predictions of the motions of the planets. Once the absolute distances in the solar system are known it is possible to take into account the pertubations of the planets' orbits which are due to gravitational interactions between the planets. Therefore, more precise predictions of the motions of the planets and, particularly, of the moon can be made.

For these reasons, the distance to the Sun is not only a scale for the solar system and, for example, for the mass of its bodies but also for the dimension of the whole universe – the so-called Astronomical Unit.

Recalling the history of that Unit and underlying basic concepts, together with experiencing some of the problems encountered during measurements, does not only provide for a better understanding of former achievements. In addition, it seems to be a suitable way to gain a deeper insight about the interplay between theory and observation in science in general, and of astronomy in particular. Calculating with data that extend far beyond men's imagination might also serve for approaching a realistic perspective on contemporary developments of science.

Figure 1: Stereoscopic picture of a landscape and the ruins of a building. You get a stereo effect by looking at the picture with the "parallel glance" so that the two black spots above the pictures merge into one.

3 The geometric parallax

If you hold an object, e.g. an apple, at arm's length, close your eyes one by one, you will observe the apple jumping from right to left and inversely in front of the distant objects in the environment. This apparent change of position, the so-called parallactic motion, is due to the changing perspective.

This parallactic effect is familiar to all of us from our daily experiences, e.g. with driving a car or travelling by train: The closer the objects in the landscape are, the faster they stay behind, that is to say in forward movement, they move backward in relative proportion to the more distant objects. The further the objects are, the smaller is the effect. That's why it can be used in order to determine the distance of objects.

This parallactic effect is familiar to all of us from our daily experiences: If we observe a landscape from different viewpoints all objects have different positions with respect to the others. The closer the objects in the landscape are, the more they change their relative positions. The further the objects are, the smaller is the effect. That's why parallax can be used in order to determine the distance of objects.

As a matter of fact, the parallaxe is a major precondition of three dimensional sight.

- Our two eyes take in different pictures in which the objects' relative positions to each other differ slightly. In our brain, the two pictures are transformed into a three dimensional picture (fig. 1).

- If the distances are too big and if, consequently, the parallactic differences are too small for a threedimensional picture, the enhancement of parallactic shifts caused by movement may help to get an impression of the spatial depth.

The parallax π of an object is the difference between the perspectives of two different observers looking at it. Or to put it differently: π is the angle by which the distance Δ of the two points of observation , e.g. the eyes or the two observatories, appear from the object's position (fig. 2). Provided that the line connecting both points of observation is perpendicular to the direction of the object, the following relation obviously holds:

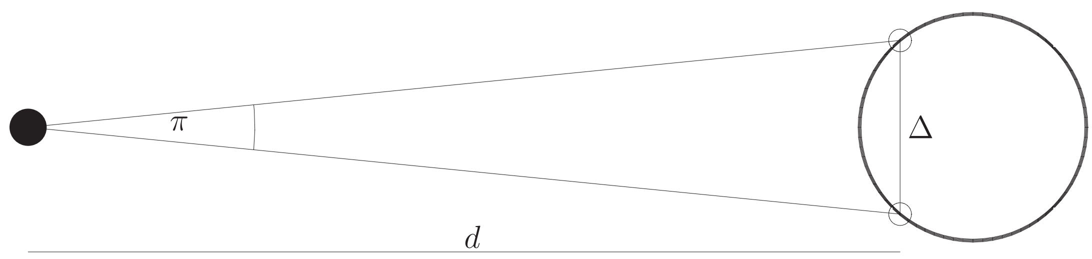

Figure 2: On the relation between parallax π, distance *Delta* between the points of observation and the distance d of the object

$$\tan\frac{\pi}{2} = \frac{\frac{\Delta}{2}}{d} \quad\Longrightarrow\quad d = \frac{\frac{\Delta}{2}}{\tan\frac{\pi}{2}} \tag{1}$$

If the distance is very large, the parallax therefore very small, the following formula can be used as an approximation:

$$\boldsymbol{d \overset{\pi \ll 1}{\approx} \frac{\Delta}{\pi}} \tag{2}$$

Until today, the measurement of so-called trigonometric parallaxes is the most precise procedure to determine the distance of astronomical objects.

> *The parallaxe of an object in the solar system is defined by the angle by which the Earth's radius appears from the object's position*[2].

If one observes the relative shift in position $\Delta\beta$ in front of an "infinitely" distant background, then it directly shows the parallaxe of the object: $\Delta\beta \equiv \pi$ (s. fig. 3[3]).

However, if one observes the parallactic shift with respect to a very, but not infinitely, distant background, the relative shift is smaller than the parallaxe because the background object itself shows a parallactic effect: $\Delta\beta = \pi - \pi_H$ (s. fig. 4).

4 The parallax of the Sun

The distance to the Sun is very large, the parallax, therefore, very small: Its value is only 8".8. That is the appearent size of a little coin in a distance of 230 m! To make it more difficult, no background can be seen when the Sun is shining. Thus, it is not possible to measure the Sun's parallax directly by a geometrical method.

[2]With regard to objects outside the solar system, e.g. the stars, the parallaxe refers to the radius of the Earth's orbit, that is to say to the distance between Sun and Earth.

[3]Figures 3 and 4 are stereoscopic pictures. You get a stereoscopic impression by looking at them with the "parallel glance" so that the two pictures seen by both eyes merge into one picture in between. The dots above the both pictures may help you: Your eyes are positioned correctly when you see one additional dot appearing exactly between both of them. It is favourable to put your eyes first very close to the pictures and then to remove them slowly until you have found the correct position.

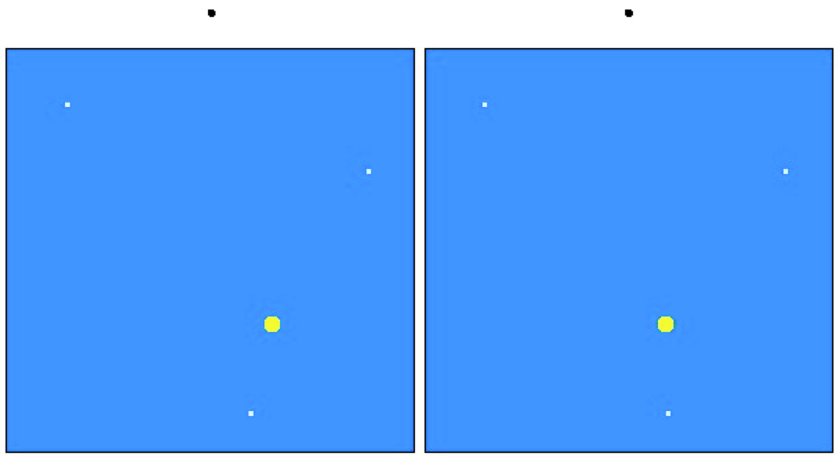

Figure 3: Parallactic shift of Venus with respect to the background of the stars. This shift equals the parallaxe of Venus π_V.

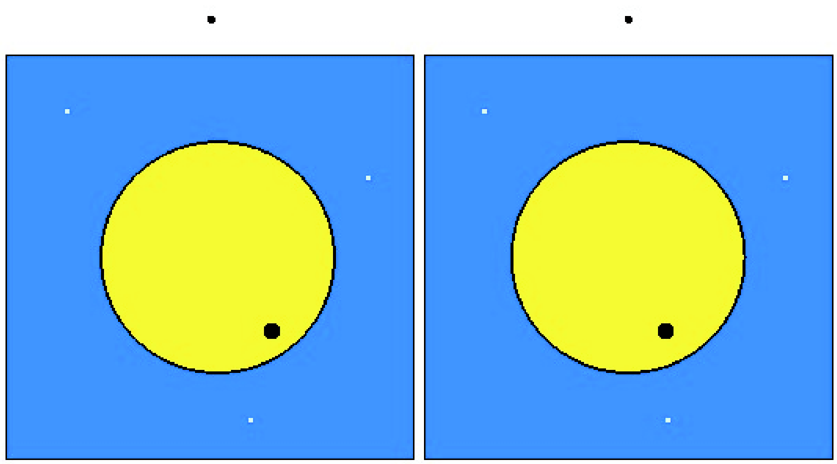

Figure 4: Parallactic shift of Venus and Sun. Because of the shift of the Sun the shift of the Venus with respect to the Sun's disc is smaller than with respect to the stars in figure 3. It corresponds to the difference $\pi_V - \pi_S$.

The basic idea of geometrically measuring the distance to the Sun is to determine the parallax of another body of the solar system and to derive from its distance that of the Sun, e.g. with the help of Kepler's third law.

The following bodies have been used:

- **Mars**, by far not as bright as the Sun and, in its opposition to the Sun, only half as far as it was the first body to translate this idea into action. Already Kepler had noticed that he could not observe any parallactic movement of Mars. From this experience, he had concluded that the distance to the Sun given by Aristarchus must be by far too small. In 1672, Cassini in Paris, Richer in Cayenne and Flamsteed in London succeeded in determining the parallactic angle of Mars to be about 25".5 and to derive a solar parallax of not more than 10".

- **Venus**, in inferior conjunction to the Sun, comes still closer to the Earth than Mars. Unfortunately, it is usually invisible in that position. But during one of the very rare transits it can be observed quite well in front of the Sun and its position with respect to the Sun's disc can be measured easily, at least in principle. Therefore, after the observation of a transit of Mercury Halley in 1716 proposed to observe and to measure the next coming transits of Venus from as many different sites on Earth as possible in order to get a measure of the distance to the Sun as precise as possible.

- Some **minor planets** come even closer to the Earth than Venus. Additionally, because of their little brightness and small size their position can be determined even more exactly. In 1931, the minor planet **Eros** led to a very exact determination of the Sun's parallax[4]. But, meanwhile it had become possible to measure it by means of physical methods.

5 Geometry of the Transit

For two observers at different sites on Earth a transit lookes slightly different: Venus enters the Sun's disc at different times and doesn't leave it simultaneously. And, taken at the same moment, Venus' position in front of the Sun is not quite the same. This parallactic effect can be observed

- if the length of Venus' path over the Sun is determined by precisely measuring the moments of ingress and egress or

- if two simultaneously taken photos are scaled to the same size and merged with exactly the same orientation (fig. 5).

How is it possible to infer the distance of Venus from Earth and finally, the distance between Earth and Sun from this parallactic shift?

[4]The determination of the distance to the Sun by measuring minor planets' parallaxes was the subject of another internet project in 1996 ([3] and [4]).

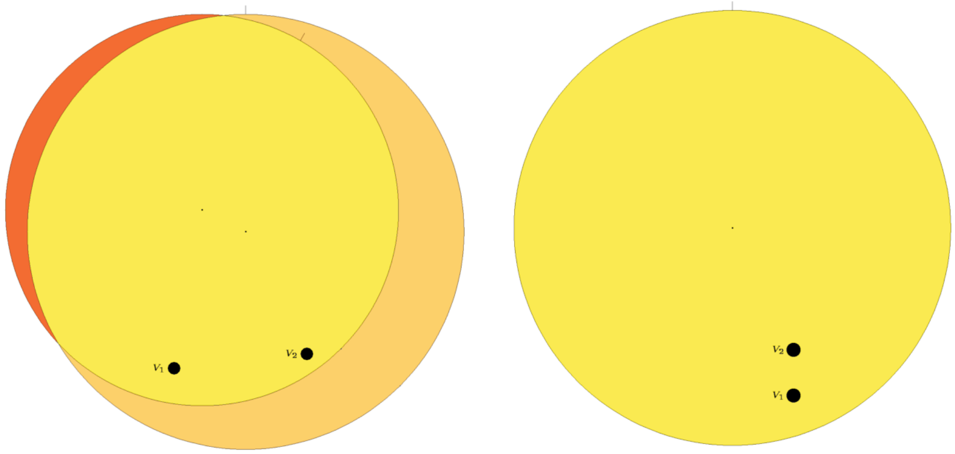

Figure 5: Two photos of Venus in front of the Sun, simultaneously taken at different sites on Earth. Left: merged arbitrarily. Right: Shifted to coincidencing centres, scaled to the same size and rotated to the same orientation. The parallactic effect is strongly exaggerated.

The parallactic shift between both discs or between both paths of transit is most often explained in the following way: Because Venus divides the distance Earth - Sun by 5:2 (see chapter 8.1) the distance between the two "projections" on the Sun must be 2.5 times as large as the separation of both observers. Therefore, the angular distance between these projections, when observed from the Earth must be 2.5 times as large as the distance of both observers appears to an observer on the Sun. In the situation shown in figure 6 the angular distance of the projections ("'shadows"') is, therefore, five times as large as the angle by which the Earth's radius is seen from the Sun, the so-called parallax π_S of the Sun.

This explanation appears to be plausible. Nevertheless, there arise some questions:

1. Why does $\Delta\beta$ represent the parallactic displacement of Venus instead of β_V?
2. Of course, on the Sun's surface there are no "shadows" of Venus. How is it, nevertheless, possible to observe their distance from Earth?
3. If it is, nevertheless, possible to see those projections by some method: Are they located on the surface or on a plane, for instance, throught the centre of the Sun? What is the orientation of that plane? Because the Sun's radius is about 0.5% of the distance between Earth and Sun, the answer to this question may be of some importance.
4. When observed from Earth the position of the Sun against the stars is a little different, too. Must this effect not be taken into account?

In fact, it is neither possible to see Venus *in front of the Sun* nor to observe "shadows" *on its surface*. Instead, only angles can be observed and measured at the celestial sphere.

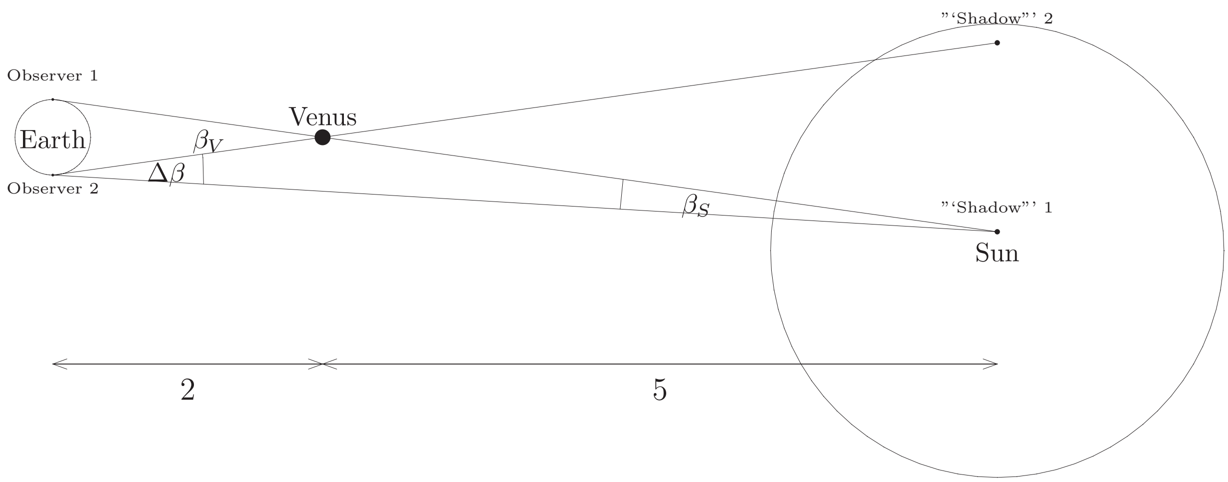

Figure 6: Usual explanation for the displacement of the two discs of Venus (e.g. Herrmann ([10])): From Earth, the distance of both "projections" of Venus is seen by the angle $\Delta\beta$. Because that distance is 2.5 times as large as the distance between the both observers, $\Delta\beta$, too, is 2.5 times as large as the angle β_S by which the Earth appears from the Sun.

Therefore, for both observers, Venus in figure 5 has different angular positions with respect to the Sun's face, e.g. different angular distances β_1 or β_2 from the centre (fig. 7)[5]. These angles can be taken from figure 5, provided the scale has been determined with the known angular diameter of the Sun. The distance of the two discs against the Sun is, therefore, the angular difference $\Delta\beta = \beta_1 - \beta_2$.

Therefore, the angle $\Delta\beta$ can not be measured absolutely but only by two measurements from different sites *with respect to the Sun's face*. In contrast to the first impression, it is not the parallax of Venus but, instead, smaler than it by the parallax of the Sun! With the remarks made in section 3 this fact is clear because the angles are not measured with respect to infinitely far stars but with respect to the Sun showing parallax by itself.

This interrelation may additionally be illustrated as follows (fig. 8): Merging the pictures of the Sun taken by two observers both discs of Venus have the distance $\Delta\beta$. But, to put the pictures of the Sun into the correct position with respect to the stars one of the pictures must be shifted by β_S. Then, β_V will be the distance of the two discs and, therefore, larger than $\Delta\beta$ by β_S.

Let β_S and β_V be the angles by which the distance of the both observers appears from the Sun and from Venus, respectively, that means the actual angles of parallax of Sun and Venus. Than, due to figure 7 the following equation holds:

$$\beta_S + \beta_1 = \beta_V + \beta_2 \tag{3}$$

That is why both sums complete the opposite angles at S to 180°, first in the triangle observer 1 - S - center of the Sun and, second, in the triangle observer 2 - S - Venus.

[5]Generally, both observers, Venus and the centre of the Sun will not be positioned in the same plane. Therefore, in fig. 5 both discs will not be located on the same radius. In that case, the two observers and Venus will define the plane of fig. 7. The argumentation remains unaltered: The difference $\Delta\beta$ is then due a point of the Sun in that plane (see also fig. 8.

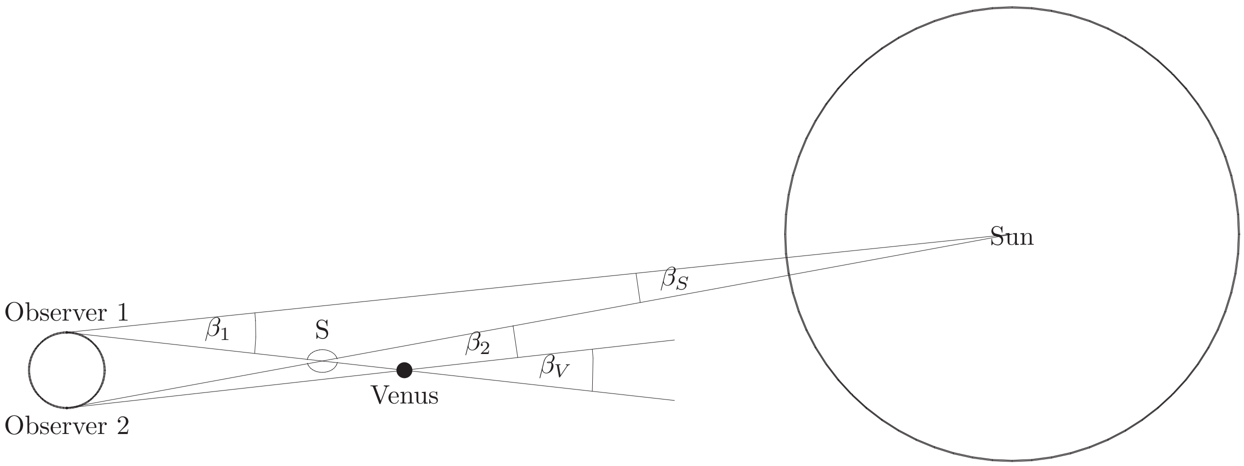

Figure 7: Alternative explanation: Different observers see Venus at different positions relative to the Sun's face. Their angular distance with respect to the Sun is $\Delta\beta = \beta_1 - \beta_2$.

That relation can be written as follows[6]:

$$\Delta\beta = \beta_1 - \beta_2 = \beta_V - \beta_S \tag{4}$$

6 Derivation of the distance to the Sun

The determination of the distance to the Sun is based on the following train of thought:

- To calculate the distance to the Sun it is necessary to know the parallax π_S of the Sun, that means the angle by which the Earth's radius appears from the Sun.
- Equally suitably is the angle β_S by which the distance of two arbitrary observers appears from the Sun provided their distance is known.
- Instead of β_S, the larger angle β_V by which the distance of the both observers appears from Venus can be measured more easily. Knowing the proportion between the distances of Sun, Venus and Earth (see chapter 8.1) one of these angles can be derived from the other.
- The angle β_V is located at Venus, but it can be derived from the angles β_1 and β_2 which can be measured from Earth.

6.1 Theory

Because the distances of Venus and Sun from the Earth are very large compared with the Earth's radius and because the corresponding parallaxes, therefore, are very small, the angles of parallax are inversely proportional to the corresponding distances d_V and d_S:

[6] The same relation may be taken from fig. 6.

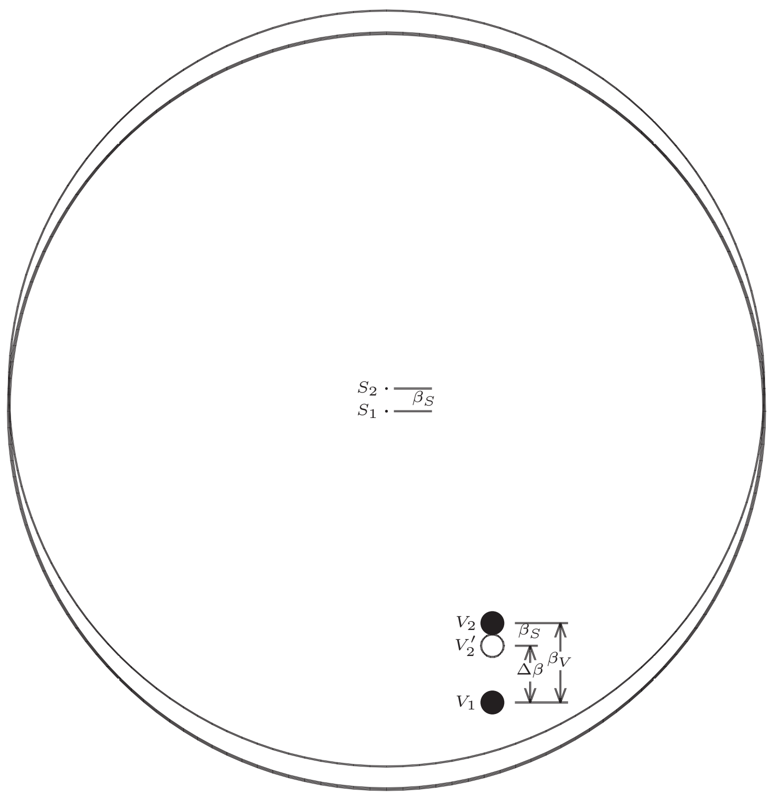

Figure 8: The pictures of the respective observers against a (fictive) background of stars: Both pictures of the Sun are shifted with respect to each other by the Sun's angle of parallax β_S, the pictures of Venus by β_V *in the same direction*. After having shifted one of the photos so that the pictures of the Sun coincide the Venus positions differ only by $\Delta\beta = \beta_V - \beta_S$.

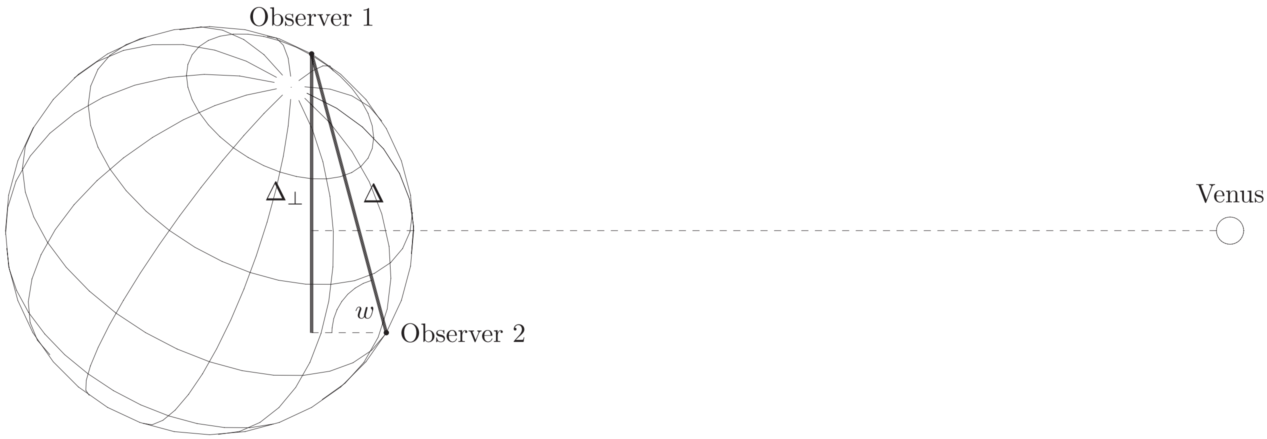

Figure 9: Not the distance Δ between the both observers itself is important for the determination of the Sun's parallax but its projection $\Delta_\perp$ parallel to the direction to Venus.

$$\frac{\beta_V}{\beta_S} = \frac{d_S}{d_V} \tag{5}$$

Let r_E and r_V be the radii of the orbits of Earth and Venus, respectively. Then the following equation follows from (4):

$$\begin{aligned} \Delta\beta &= \frac{r_E}{r_E - r_V}\beta_S - \beta_S = \frac{r_V}{r_E - r_V}\beta_S \\ \Longrightarrow \quad \beta_S &= \left(\frac{r_E}{r_V} - 1\right)\Delta\beta \end{aligned} \tag{6}$$

In practice, the angle $\Delta\beta$ is not measured absolutely but as a fraction f of the angular radius ρ_S of the Sun:

$$\Delta\beta = \frac{\Delta\beta}{\rho_S}\rho_S = f\rho_S \tag{7}$$

In the special case shown in fig. 6 and 7 the distance between the observers is twice as large as the radius of the Earth and, therefore, the angle of parallax β_S is twice the parallax of the Sun π_S which is related to the Earth's radius. In general, one must know the distance Δ between the observers as a multiple of the Earth's radius, to be more precisely: the projected distance $\Delta_\perp$ of the two observers *perpendicular* to the direction to Venus (fig. 9).

Therefore, the following equations must hold

$$\beta_S = \pi_S \frac{\Delta_\perp}{R_E} = \pi_S \frac{\Delta}{R_E}\sin w \quad \Longrightarrow \quad \pi_S = \frac{R_E}{\Delta}\frac{1}{\sin w}\beta_S$$

and, finally,

$$\boldsymbol{\pi_S = \left[\frac{R_E}{\Delta} \frac{1}{\sin w} \left(\frac{r_E}{r_V} - 1 \right) \rho_S \right] f.} \tag{8}$$

From this result for the parralax of the Sun π_S, the distance to the Sun, the so-called Astronomical Unit, can be derived by the following equation (see (2)):

$$\boldsymbol{1AE = d_S = \frac{R_E}{\pi_S}} \tag{9}$$

6.2 An Example

Two pictures of the transit of Venus on June 6th, 1761 may be given, simultaneously taken from Koblenz, Germany, and Windhoek, Namibia, respectively, at 7.00 UT ([24]).

1. On these pictures, the distance between the both discs of Venus can be measured to be quite exactly 2.6% ($f = 0.026$) of the radius of the sun. On that day, the angular radius of the sun was $\rho_S = 15'.75$. Therefore, the angular distance $\Delta\beta$ of the two discs is

$$\Delta\beta = f\rho_S = 24''.57$$

2. On that day, the relative distance between Earth and Sun was $r_E = 1.015AE$, the distance between Venus and Sun $r_V = 0.711AE$ ($\frac{r_E}{r_V} - 1 = 0.428$).

   According to equ. (6), from the Sun the angular distance between the both cities is

$$\beta_S = \left(\frac{r_E}{r_V} - 1 \right) \Delta\beta = 10''.51.$$

3. The geographical coordinates of the cities are[7]:

   Koblenz: $\varphi_K = 50.24°$, $\lambda_K = 7.36°$,

   Windhoek: $\varphi_W = -22.34°$, $\lambda_W = 17.05°$.

   Transforming these polar coordinates in rectangular coordinates and calculating the length of the connecting vector one gets:

$$\Delta = 1.19 R_E \quad \Longrightarrow \quad \frac{R_E}{\Delta} = 0.840$$

4. If the line Koblenz - Windhoek were perpendicular to the line Earth - Sun the parallax of the Sun would be, due to (8),

$$\pi_S \overset{w=90°}{=} \frac{R_E}{\Delta} \beta_S = 8''.83.$$

[7] In this paper, northern latitude and eastern longitude are taken positive.

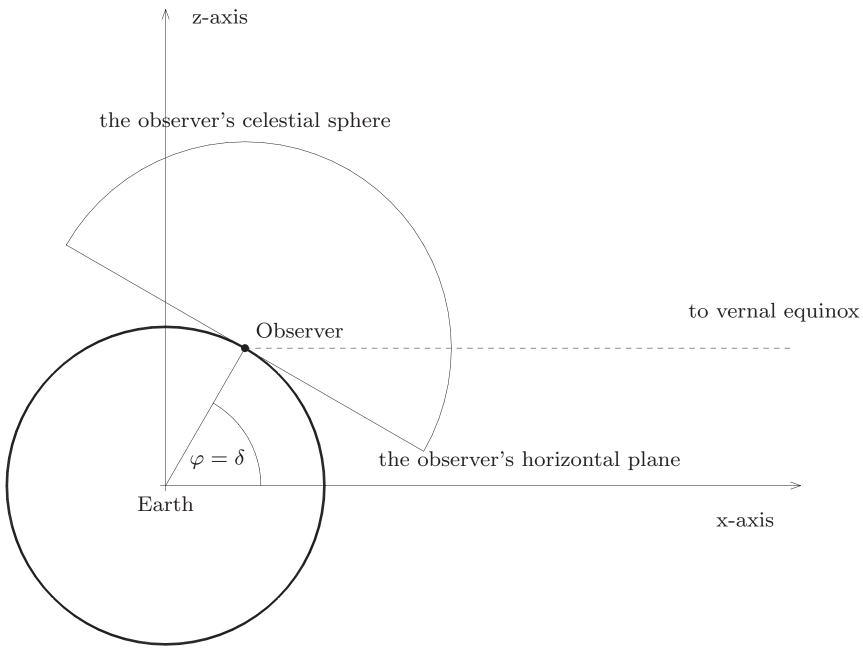

Figure 10: Determination of the geocentric equatorial coordinates of the observers (see [25]): If the local sideral time is 0h, the vernal equinox, therefore, is just culminating, the right ascension o the observer is 0h, too.

5. In order to be able to determine the angle w the coordinates of the Earth, the Sun and of the both cities must be known *with respect to the same coordinate system.* For this reason, we take geocentric equatorial coordinates.

   On June 6th, the position of the Sun is:

   $$\alpha_S = 4h57min28s \stackrel{\wedge}{=} 74.37°,\ \delta_S = 22°39'.6 = 22.66°.$$

   The declination of the cities equals their geographical latitude (s. Abb. 10). Their right ascension equals their local sideral time (s. [25]). It can be calculated as follows:

   (a) On June 6th, 1761 at 0.00h UT the local sideral time of Greenwich was[8]

   $$\Theta_{0_{Gr}} = 16h58min24s.$$

   (b) Therefore, at 7.00 UT the sideral time of Greenwich was[9]

   $$\Theta_{Gr} = \Theta_{0_{Gr}} + 7.00 * 1.0027379 = 23h59min33s.$$

[8]It can be determined by some simple methods, e.g. by counting the days which have passed since the beginning of springtime when UT and local sideral time of Greenwich differed just by 12h. With sufficient accuracy, on a certain date the sideral time is the same every year. Therefore, it can be taken from an actual astronomical almanach.

[9]The factor 1.0027379 is due to the fact that, during 24 hours, the Earth rotates more than once and, therefore, more than 24 h sideral time pass.

(c) At an arbitrary site of longitude λ, the sideral time at the same moment is

$$\Theta = \Theta_{Gr} + \frac{4min}{1^\circ}\lambda.$$

The local sideral times of Koblenz and Windhoek, therefore, were:

$$\Theta_K = 0h28min59s,\ \Theta_W = 1h07min45s$$

(d) Therefore, the equatorial coordinates of the both observers are
Koblenz: $\alpha_K = 7.246^\circ,\ \delta_K = 50.24^\circ$,
Windhoek: $\alpha_W = 16.938^\circ, \delta_W = -22.34^\circ$.

(e) Transforming the positions to rectangular coordinates and determining the vector connecting Koblenz and Windhoek one gets the angle w by calculating the scalar product of the corresponding unit vector $\vec{e}_{KW}$ and the unit vector $\vec{e}_S$ pointing to the Sun. In this way, one gets

$$\vec{e}_S \cdot \vec{e}_{KW} = \cos w = -0.179 \quad \Longrightarrow \quad w = 100.28^\circ \quad \Longrightarrow \quad \sin w = 0.984.$$

Therefore, the projected distance between the both cities is $\Delta_\perp = 1.175R_E$.

(f) Putting all these results into equ. (8) we get the final result:

$$\boldsymbol{\pi_S = 8''.97}$$

7 Measurements

The equations (8) and (9) summarize how it is possible to get a value for the distance to the Sun by observing and measuring a transit of Venus. They make clear what must be measured and what must be calculated in order to determine the parallax of the Sun.

- The basic measure is the **angular distance $\boldsymbol{\Delta\beta}$** of two discs of Venus on the Sun's face, the positions of which are measured simultaneously from different sites on Earth with respect to the Sun's disc. First, that angular distance is determined as a **fraction $\boldsymbol{f}$** of the Sun's angular radius.

- In order to transform $\boldsymbol{f}$ into an absolute angle the **angular radius $\boldsymbol{\rho_S}$** of the Sun must be determined.

- In order to infer the parallax of the Sun from the actual parallax angle $\boldsymbol{w}$, the **linear distance $\boldsymbol{\Delta}$** of the both observers must be determined as a **multiple $\boldsymbol{\frac{\Delta}{R_E}}$** of the Earth's radius. For this reason, the **geographical coordinates $\boldsymbol{(\varphi_i, \lambda_i)}$** of the observers must be known.

- Not the distance of the observers itself is of importance but its projection parallel to the direction Earth - Sun. Therefore, the **angle of projection $\boldsymbol{w}$** must be determined. For that, it is necessary to determine the time of observation

- the **equatorial coordinates** $(\boldsymbol{\alpha_S}, \boldsymbol{\delta_S})$ of the Sun and
- the **local sideral times** $\boldsymbol{\theta_i}$ of the both sites of observation.

- In order to conclude from Venus' angle of parallax β_V to that of the Sun β_S the **radius of Venus' orbit** must be known in relation $\frac{r_V}{r_E}$ to the Earth's radius.
- Finally, to be able to derive the distance to the Sun in absolute terms one must know the value of the **Earth's radius** $\boldsymbol{R_E}$.

8 The internet project "Venus 2004"

In 2001, U. Uffrecht ([19], [20]) have called for an international project which is meanwhile coordinated from Essen in Germany. It is the main objective of this project to bring into contact school classes, astronomical work groups, groups of astronomical amateurs and observatories in order to corporately observe and photograph the transit of Venus in 2004 and to derive the distance to the Sun from own data by different methods. Afterwards, the material will be arranged so that it will offer possibilities for evaluation with different claims for exactness in education.

> ***The months before the transit are used as a comprehensive educational project. Within the framework of the developing international cooperation all quantities which are explicitly or implicitly contained in equations (8) und (9)***
>
> $$\begin{aligned} \boldsymbol{\pi_S} &= \left[\frac{\boldsymbol{R_E}}{\boldsymbol{\Delta}}\frac{\mathbf{1}}{\sin \boldsymbol{w}}\left(\frac{\boldsymbol{r_E}}{\boldsymbol{r_V}} - \mathbf{1}\right)\boldsymbol{\rho_S}\right]\boldsymbol{f} \\ \mathbf{1}\boldsymbol{AE} &= \boldsymbol{d_S} = \frac{\boldsymbol{R_E}}{\boldsymbol{\pi_S}} \end{aligned}$$
>
> ***will be determined by own measurements.***

To obtain this goal the following preparation projects have been created:

1. Measuring the radius of Venus' orbit (chapter 8.1, S. 17)
2. Determining of the own geographical coordinates and the projected distance of different observers (chapter 8.2, S. 24)
3. Determining the radius of the Earth (chapter 8.3, S. 24)
4. Measuring the angular radius of the Sun (chapter 8.4, S. 24)
5. Exercises in photographing the Sun and exact position measurements on the Sun's disc (Sunspots) (chapter 8.6, S. 24)
6. The Transit of Mercury on May 7th, 2003 (chapter 8.7, S. 24)

8.1 The radius of Venus' orbit

To be able to derive an own measure of the Astronomical Unit from the self determined value of the parallax of Venus by equation 8, among other quantities, the radius r_V of Venus' orbit must be known or, to be more precise, the relation $\frac{r_V}{r_E}$ between the radii of the orbits of Venus and the Earth. There are several possibilities for measuring this quantity:

1. Measuring the maximum angular distance between Sun and Venus both of them above the horizont at the same time by means of a simple device for angle measurements or, alternatively, with a sextant.

2. Other methods of determining the maximum angular distance between Sun and Venus, for instance

   (a) observing Venus at the night sky, recognizing her position relative to stars in the neighborhood and determining her equatorial position by drawing her into a star map and

   (b) measuring the exact horizontal position of the Sun and transforming her horizontal to equatorial coordinates (see chapter 8.5).

   (For this method, you need to know how to determine the local sideral time and your own geographical position.)

3. Observing Venus during her retrograde motion and measuring her equatorial positon at least two times.

All of these methods make use of the following simplifications:

- ***All planets move in the same plane as the Earth.***
- ***All orbits are circles with the Sun in their center.***
- ***All planets move with constant speed.***

8.1.1 The largest angular distance to the Sun

When the angular distance between Sun and Venus, observed from Earth, has its maximum value the triangle Sun - Venus - Earth is rectangular.

Therefore, the radius of Venus' orbit, in Astronomical Units, can easily be determined by measuring the maximum angular distance between Sun and Venus:

$$\sin\psi_{max} = \frac{r_V}{r_E} \quad \Longrightarrow \quad r_V = \sin\psi_{max} AU \tag{10}$$

The simplest method to measure the distance and to determine its maximum is to observe both objects above the horizont, simultaneously. If you observe Venus at the night sky early in the nigh you will be able to find her shortly after sunrise.

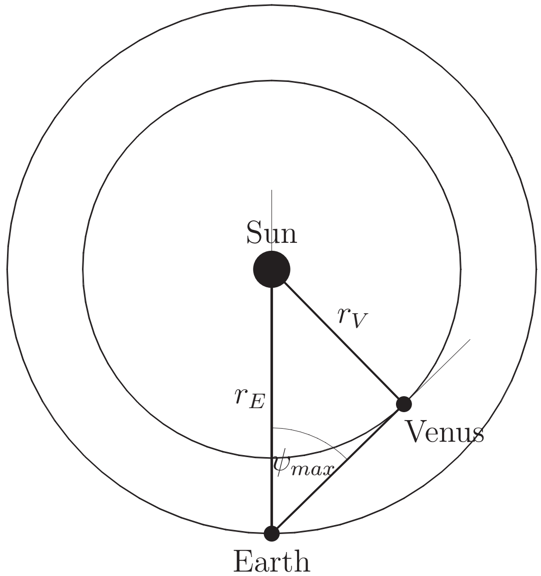

Figure 11: Maximum elongation of Venus and the radius of its orbit

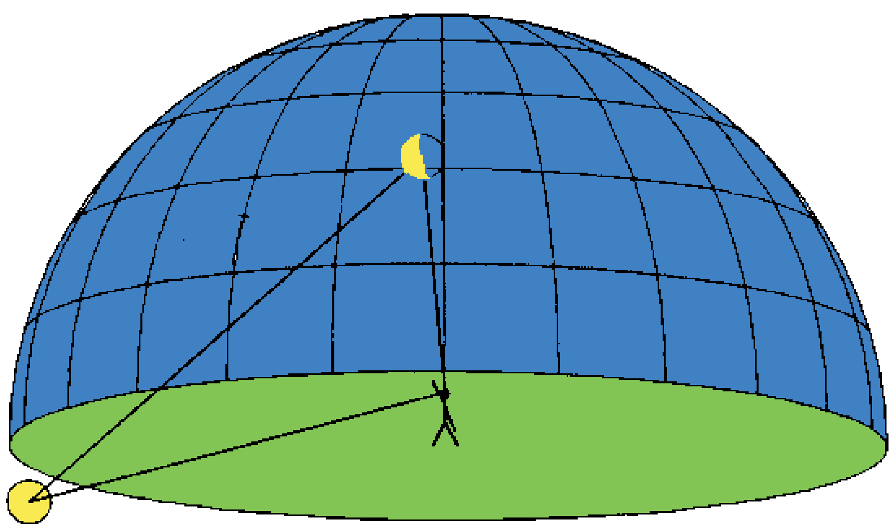

Figure 12: Measuring the elongation of Venus

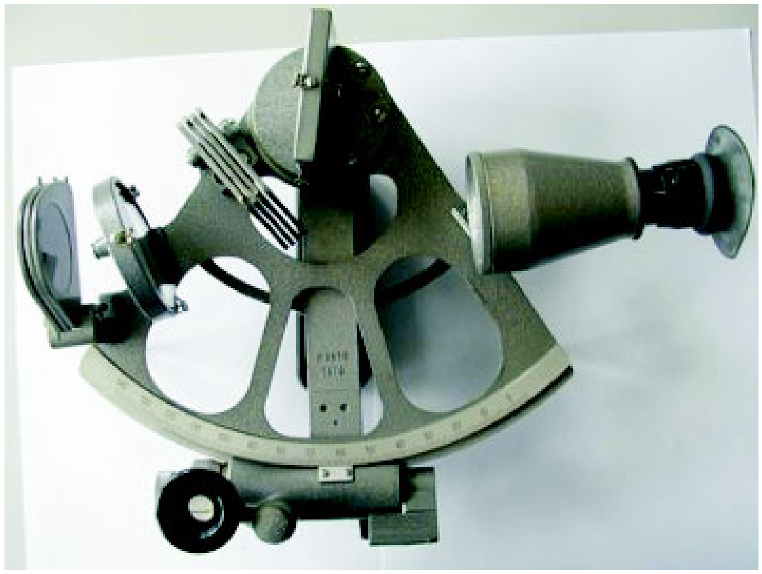

Figure 13: A sextant is a device for precise angular measurements.

Proposals for rough measurements of the angle[10]:

- Mark the lines of sight with pins on a piece of carton.
- Use a pair of compasses.

It is possible to find other simple possibilities.

A more precise method is to use a sextant, a device which was invented just for this kind of measurements.

8.1.2 Venus' retrograde motion

Caution: This method can be applied only in a short time interval around the transit, that is between May 23th and June 28th!

Around the inferior conjunction of Venus, at the sky the planet will move retrogradely (fig. 14), that means opposite to its usual direction (from east to west). That's why Venus overtakes the Earth in that time. Figure 15 gives an explanation for this fact not only in a qualitative way but in a quantitative, too.

In the triangle Sun - E_1 - P_1, the following equation holds:

$$\frac{r_P}{r_E} = \frac{\sin\mu}{\sin(\eta+\beta)}$$

Looking at the triangle Sun - E_1 - S the following identity can be realized:

[10] You may get an impression of the measurements on our first result page `http://didaktik.physik.uni-essen.de/~backhaus/Venusproject/orbitresults.htm.`

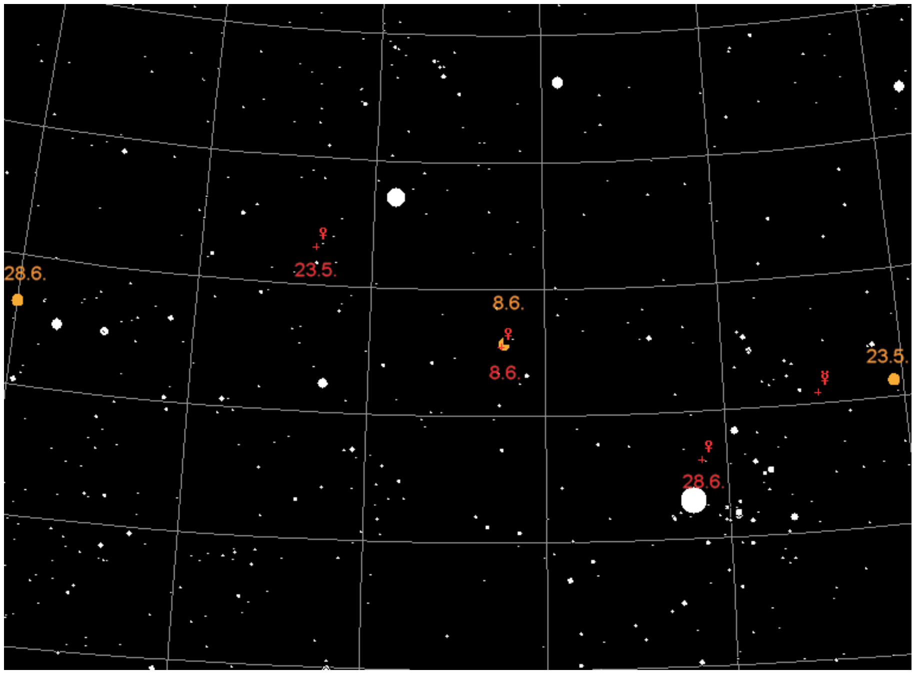

Figure 14: Venus' positions at the beginning and at the end of its retrograde motion

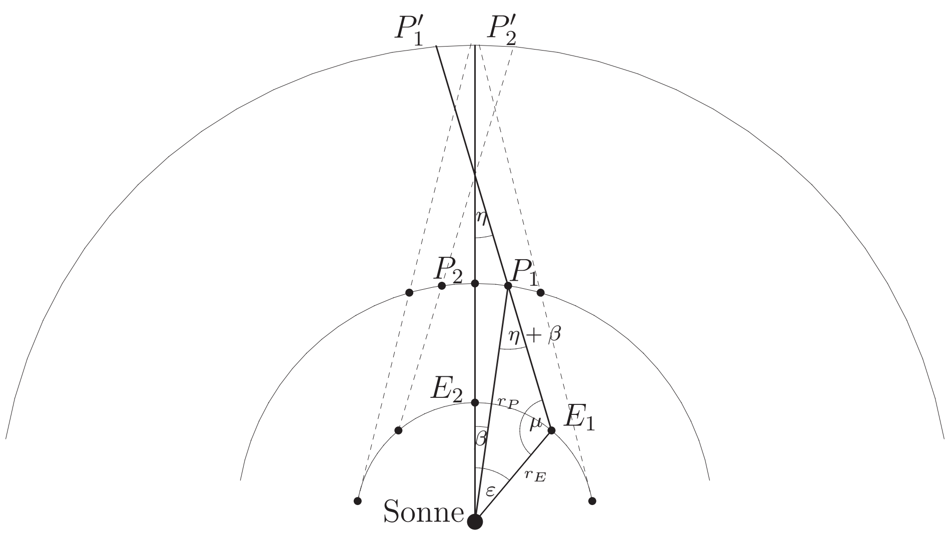

Figure 15: Explanation of the retrograde motion of a superior planet. In the case of Venus which is an inferior planet the letters P and E must be interchanged. The argumentation remains unaltered.

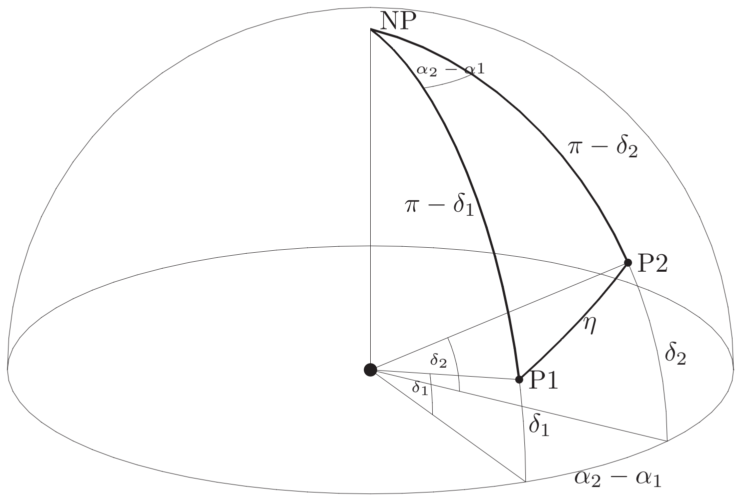

Figure 16: Derivation of the angular distance between two celestial bodies from their equatorial coordinates

$$\mu = \pi - (\epsilon + \eta) \quad \Longrightarrow \quad \sin\mu = \sin(\epsilon + \eta)$$

Combining the both equations, one gets the following result:

$$\frac{r_V}{r_E} = \frac{\sin(\epsilon + \eta)}{\sin(\beta + \eta)} \quad \Longrightarrow \quad r_V = \frac{\sin(\epsilon + \eta)}{\sin(\beta + \eta)} AU$$

We hope that we will be able to observe and to measure Venus's position in its inferior conjunction, that is during its transit! Thus, we can measure its right ascension and declination on that day (see chapter 8.5). If we will be able to measure Venus' position on an additional day during its retrograde motion we can determine, according to figure 16, the angular distance η between these both positions by the following relation:

$$\cos\eta = \sin\delta_{V_1}\sin\delta_{V_2} + \cos\delta_{V_1}\cos\delta_{V_2}\cos(\alpha_{V_1} - \alpha_{V_2})$$

The central angles at the Sun can be derived from the mean daily motion of the planets which are due to their sideral periods:

$$\beta = \mu_E \Delta t = \frac{2\pi}{365.256d}\Delta t, \ \epsilon = \mu_V \Delta t = \frac{2\pi}{224.70d}\Delta t$$

Example:

1. On June 8th, during the transit Venus has the following position:

$$\alpha_{V_1} = 5h07m, \ \delta_{V_1} = 22.88^\circ$$

2. On June 28th, the position of Venus is:

$$\alpha_{V_2} = 4h34m,\ \delta_{V_2} = 18.05^\circ$$

3. During those 20 days, Venus, therefore, has apparently moved by

$$\eta = 9.05^\circ.$$

4. During that time interval, Earth and Venus have passed the following central angles:

$$\beta = 19.71^\circ,\ \epsilon = 32.04^\circ.$$

5. Taking these results together, we find the following radius of Venus' orbit:

$$r_V = 0.73AU$$

8.1.3 Generalisation

If the transit can not be observed the method, nevertheless, can be applied by measuring Venus' position during its retrograde motion twice. In this case, the following equation must hold:

$$\frac{\sin(\epsilon_1 + \eta_1)}{\sin(\beta_1 + \eta_1)} = \frac{\sin(\epsilon_2 + \eta_2)}{\sin(\beta_2 + \eta_2)} = \frac{\sin(\epsilon_2 + \eta_1 + \eta)}{\sin(\beta_2 + \eta_1 + \eta)}$$

In this equation, the single angles η_1 and η_2 are unknown. Only the angle $\eta = \eta_2 - \eta_1$ has been measured. The central angles are again determined with respect to the date of conjunction.

The following function f must, therefore, be zero at the point η_1:

$$f(\eta_1) = \sin(\epsilon_1 + \eta_1)\sin(\beta_2 + \eta_1 - \eta) - \sin(\epsilon_2 + \eta_1 - \eta)\sin(\beta_1 + \eta_1) = 0$$

This is a well known numerical problem and can be solved by trial and error, by a graphical method or by a numerical algorithm, respectively. You may download a little program[11] which does the work for you.

Example

1. Suppose you have measured Venus' position on May 23th, instead of June 8th:

$$\alpha_{V_1} = 5h40m,\ \delta_{V_1} = 26.43^\circ$$

2. Then, the numerical determined zero of the above function is:

$$\eta_1 = 8.99^\circ$$

3. From this value follows the same radius as in the above example.

[11] `http://didaktik.physik.uni-essen.de/~backhaus/Venusproject/RadiusofOrbits.exe`

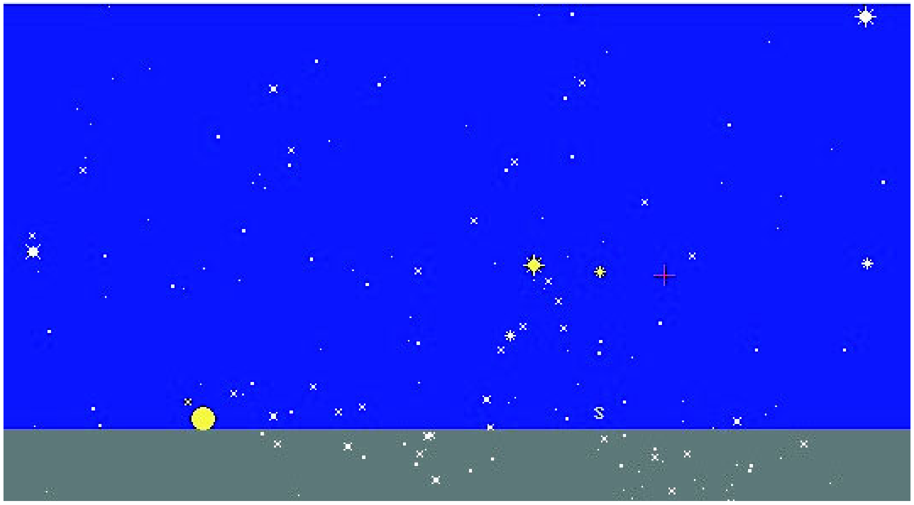

Figure 17: Position of Venus and Mars on January 11th, 2003

8.1.4 The equatorial coordinates of Venus

The easiest method of determining the celestial position (α, δ) of Venus is to observe her exact position among the stars before sunrise or after sunset, respectively, and draw her into a sky map (see figure 17) from which you can read the equatorial coordinates.

Because Venus is often quite close to the Sun the sky is not dark enough to locate Venus among the stars. In this case, it is necessary to determine her celestial position by determining her position with respect to the local horizont. For this method, the **azimut** A has to be measured, for instance with a magnetic compass, and the **elevation** h above the horizont and the exact time of your measurement. From this values, the equatorial coordinates can be derived by the following algorithm:

$$\left.\begin{aligned} a = \cos\tau\cos\delta &= \cos A\cos h\cos\varphi + \sin h\cos\varphi \\ b = \sin\tau\cos\delta &= \sin A\cos h \\ c = \sin\delta &= -\cos A\cos h\cos\varphi + \sin h\sin\varphi \end{aligned}\right\} \Longrightarrow$$

$$\Longrightarrow \quad \left\{ \tan\frac{\tau}{2} = \frac{1-\cos\tau}{\sin\tau = \frac{\cos\delta - a}{b}} \right. \quad \Longrightarrow \quad \begin{aligned} \delta &= \arcsin c \\ \tau &= 2\arctan\frac{\cos\delta - a}{b} \end{aligned}$$

From the hour angle τ, the right ascension α can be derived the local sideral time θ (see chapter 8.2.3) is known:

$$\alpha = \theta - \tau$$

8.2 Geographical coordinates and the projection parallel to the direction to Venus

8.2.1 Geographical latitude and longitude

8.2.2 Projected distance

8.2.3 Local sideral time

8.3 The Earth's radius

8.4 The angular radius of the Sun

8.4.1 The Size of pin hole pictures of the Sun

8.4.2 Daily motion of the Sun

8.5 The equatorial Position of the Sun

8.6 Position measurements on the Sun's face

8.6.1 The orientation of the Sun's face

8.7 Transit of Mercury in 2003

8.7.1 Distance between simultaneously taken pictures of Venus

8.7.2 The length of Venus' pass across the Sun's face

9 Calculations

References

[1] Ball, R., *A Treatise on Spherical Astronomy*, Cambridge University Press: Cambridge 1908

[2] Homepage des Venusprojektes: `http://didaktik.physik.uni-essen.de/~backhaus/VenusProject.htm`

[3] Astronomy Online: *Measuring the Distance to the Sun*, `http://www.eso.org/outreach/spec-prog/aol/market/collaboration/solpar/`

[4] Astronomy Online: *Measuring the Distance to the Sun – Final Report*, tt http://didaktik.physik.uni-essen.de/∼backhaus/aol/finalrep.htm

[5] U. Backhaus: *Die Entfernung der Sonne*, Astronomie und Raumfahrt 35/1, 30 (1998)

[6] Backhaus, U., *Computerprogram "Finsternis"*, kann gegen Einsendung einer formatierten Diskette und eines frankierten DinA5-Rückumschlages bezogen werden.

[7] Gottwald, M., *Venusdurchgänge und die Astronomische Einheit*, Staatsexamensarbeit, Koblenz 1996.

[8] Guckelsberger, K., *Der vermessene Himmel*, Physik in unserer Zeit 32/3, 128 (2001)

[9] A. v. Helden: *Measuring the Universe*, The University of Chicago Press, Chicago 1995

[10] D. B. Herrmann: *Kosmische Weiten*, Barth: Leipzig 1977, p. 46ff

[11] M. Hunter: *Transit of Venus*, Blizzard Publishing Ltd. 1992

[12] A. Lothian:*The Transit of Venus*, Granta Books 2002

[13] Newcomb, S., *Discussion of Observations of the Transits of Venus in 1761 and 1769*, in: Selected Papers Vol. II, Bureau of Equipment, Navy Department: Washington 1891

[14] E. Maor: *June 8, 2004 – Venus in Transit*, Princeton University Press: Princeton 2000

[15] Peter, B., *Untersuchung des Vorüberganges der Venus vor der Sonnenscheibe im Jahre 1882*, in: Verhandlungen der Leopoldinisch-Carolinisch Deutschen Akademie der Naturforscher, Band 39: Dresden 1877

[16] D. Sellers, *The Transit of Venus*, Magavelda Press: 2001

[17] D. Sobel, *Längengrad*, Berlin Verlag: Berlin 1996.

[18] Stapelberg, J., *Planetenvorübergänge – Ein Beispiel für die Messung der Astronomischen Einheit*, Staatsexamensarbeit, Osnabrück 1995.

[19] U. Uffrecht, *Aufruf zu einem weltumspannenden Unterrichtsprojekt*, Astronomie und Raumfahrt 38/2, 18 (2001)

[20] U. Uffrecht, *Die Messung der Astronomischen Einheit*, Sterne und Weltraum 8, 656 (2001)

[21] S. Webb: *Measuring the Universe*, Springer: London etc. 1999, p. 44ff

[22] Wolf, R.: *Handbuch der Astronomie ihrer Geschichte und Litteratur*, Zürich 1890-1891, nachgedruckt bei Olms: Hildesheim 1973.

[23] Woolf, H.: *The Transits of Venus – A Study of Eighteenth-Century Science*, Princeton University Press: Princeton 1959.

[24] The Transit of Venus June 8th, 2004: Problems for Preparation, `http://didaktik.physik.uni-essen.de/~backhaus/VenusProject/Problems.htm`

[25] Measuring the Distance to the Sun: Mathematical Details, `http://www.eso.org/outreach/spec-prog/aol/market/collaboration/solpar/solpar-det.html`